ON THE DEVELOPMENT OF A DIVORCE INTENSITY FUNCTION

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The nature of divorce as a function of marriage duration is virtually unassessed. The purpose of this paper is to examine divorce statistics in much the same way that one might look at mortality. Just as the intensity of mortality is varying at each moment of age, the intensity of divorce is varying at each moment of marriage. Therefore, part of this paper deals with measuring this instantaneous variation. After these estimates of divorce intensity were found for each interval of marriage, a smooth curve was fitted. This curve, a hazard function for divorce, was then used to attain conditional probabilities of divorce for duration of marriage intervals. This paper is believed to be the first attempt to develop and use a divorce intensity function. Nineteen Seventy-One is used as the base year for the study as it was the most recent year for which the necessary vital statistics data were available.

As the method of the present study is similar to a method used to obtain mid-interval estimates of the "force of mortality", this method will be reviewed. A typical mid-interval estimate of the mortality intensity or "force of mortality", μ_i, for a population of N_i people of exact age x_i subject to death in the interval $(x_i, x_i + n_i)$ for a given year is given by:

$$M_i = D_i/\{n_i(N_i - D_i) + a_i n_i D_i\} \qquad \{1\}$$

where M_i is the age specific death rate, D_i is the number of deaths in the interval, n_i is the interval width, and a_i is the fraction of the interval $(x_i, x_i + n_i)$ before death occurs. The denominator of {1} is usually estimated by the midyear population, P_i, which is obtained from the Bureau of the Census (2). Thus, the age-specific death rate is the ratio of the deaths in an interval to the average number of individuals exposed to the risk of death.

A mid-interval estimate of divorce intensity, directly analogous to the age-specific death rate was developed for a population of N_i couples whose marriages endured x_i years. These couples are subject to divorce in the interval $(x_i, x_i + n_i)$, where x_i is the number of years married and n_i is the width of the ith interval.

The estimate is given by:

$$D_i = T_i \,/\, \{.5(N_i' + N_i' - T_i - L_i)\} \qquad \{2\}$$

where D_i is the duration of marriage-specific divorce rate, T_i is the number of divorces in 1971 among couples married $x_i - x_{i+1}$ years as of 1971, L_i is the number of marriages $x_i - x_{i+1}$ years prior to 1971 ended by death of a spouse during 1971, N_i' is $N_i - E_i - M_i$ which is the number of couples married $x_i - x_{i+1}$ years prior to 1971, exposed to the risk of divorce at the beginning of 1971, N_i is the number of couples married $x_i - x_{i+1}$ years prior to 1971, E_i is the number of marriages $x_i - x_{i+1}$ years prior to 1971 ended by divorce prior to 1971 and M_i is the number of marriages $x_i - x_{i+1}$ years prior to 1971 ended by death of a spouse prior to 1971. Since $N_i' - T_i - L_i$ represents the number of cohort couples still married by the end of 1971, D_i is the ratio of divorces in a duration of marriage interval to the average number of couples exposed to the risk of divorce. Thus, for each duration of marriage interval, $D_i = \hat{\delta}_i$. $\hat{\delta}_i$ represents the divorce intensity value after $x_i + n_i/2$ years of marriage.

T_i was obtained by multiplying the number of divorces granted in 1971, 764,000, by percentages given in Table 2-4 (3). These percentages were based on the divorce registration area which consisted of samples taken in 29 states. N_i represents the number of marriages $x_i - x_{i+1}$ years prior to 1971, for i = 1,2, ..., 10, x_i and x_{i+1} differ by one year, and for i = 11, 12, ..., 14, x_i and x_{i+1} differ by five years. Values of N_i, i = 2,3, ..., 10, were developed by considering the 12 months of 1971 in which a divorce could have occurred. Examination of recent data indicated that the number of divorces in each month is rather constant and consequently divorces in each of the 12 months were considered equally likely. A couple divorced in any month in 1971 after x_i years of marriage could have been married x_i years and zero months to x_i years and 11 months prior to the month of their divorce. Thus, considering these time intervals for each of the 12 months, there were nine 23 month intervals $x_i - x_{i+1}$ years prior to 1971 over which couples exposed to the risk of divorce in 1971 could have been married. For example, couples married 3-4 years in 1971 could have been married in any of the months of the interval (February, 1967, December 1968). A weighting scheme for each of the 23 months was developed based on the number of times each month was a possible marriage month. N_2 through N_{10} were determined using the weights with marriage data for each of the 23 months. The weights for the 23 months sum to 12. 1970 and 1971 marriages were averaged to obtain N_1, marriages 0-1 years prior to 1971.

N_{11}, N_{12}, N_{13}, and N_{14} represent marriages over a five year period. There are 71 possible marriage months involved for each of these N_i values. For example, the months for N_{11} (10 years 0 months - 14 years, 11 months of marriage) are February 1956-December 1961. The numerators of the weights ranged from one to 12 and each denominator was 12, so that the sum of the weights was 60. No marriage by month data was available prior to 1949. For months in years prior to 1949, the 1949 data was used. For N_{15}, the number of couples married 30-45 years prior to 1971, the marriages in the years 1926-1940 were added. Examination of data at the time where divorces were recorded in the interval 40-45 years of marriage revealed significantly large enough numbers to allow for the possibility of divorce up to 45 years of marriage. The number of divorces beyond 45 years was considered negligible.

E_i, divorces prior to 1971 among couples married $x_i - x_{i+1}$ years as of 1971 was obtained by applying percent divorces by duration of marriage in the registration area from 1926-1970 to the total number of divorces for each of these years. It was assumed that a divorce within the first 6 months of marriage was impossible. To demonstrate a typical calculation, E_4 is used. Couples married 3-4 years prior to 1971 were married in either 1967 or 1968. These couples

could have been divorced in 1967 or 1968 before their marriage had endured one year. There were 30,334 divorces in 1967 and 29,764 divorces in 1968 among couples married less than one year. A weighting scheme, too detailed to describe here, yielded 29,899 as the appropriate figure for the divorces within one year of marriage. There were 54,954 divorces in 1969 among couples married 1-2 years. There were 66,552 divorces in 1970 among couples married 2-3 years. Therefore, E_4 =29,899 + 54,954 + 66,552 = 151,405.

To obtain M_i values, a force of mortality function of the form $\mu_x=ax^b\exp(cx)$ developed by Keats and Como (1) and based on 1970 data was used to obtain $q(t_1,t_2) = 1-\exp(-\int_0^{t_2} {}_x dx)/\exp(-\int_0^{t_1} \mu_x dx)$. {3}

$q(t_1,t_2)$ represents the probability of death in an interval given survival prior to the interval. For each i, the N_i value was used with Table 1 to determine the approximate number of brides and grooms in each age category.

Table 1: Age at time of marriage based on averages of years 1962-1971. From (3):

Age	Percent Brides	Percent Grooms
15 - 20	36	14
20 - 25	36	46
25 - 30	10	16
30 - 35	5	7
35 - 45	6	8
45 - 65	6	7
≤ 65	1	2

{3} was then used with these values on a year to year basis from the year of marriage until 1971. {3} was first applied to the males for a one year period. This yielded the number of married males in each of seven categories dying within one year of marriage. Table 2 was used to identify and remove from the female population, the resulting widows in each age category. {3} was then applied to the females for a one year period and the number deceased in each age category was identified. Use of Table 2 then removed the resulting widowers from each age category. One year was added to the ages of the survivors in each age category and the process was repeated through 1970. The corresponding figure for 1971, L_i, was developed by extending the procedure one additional year.

Table 3 presents values of the statistics T_i, N_i, E_i, M_i, N_i', L_i, and D_i for fifteen marriage duration intervals. The D_i values are mid-interval estimates of δ_i, the divorce intensity value. These fifteen D_i values were plotted against duration of marriage mid-intervals ($x_i + n_i/2$) and an attempt was made to fit them with a continuous curve. Several functions were applied to these fifteen points, the best of which was the form

$$\delta_x = ax\exp(b\sqrt{x}). \qquad \{4\}$$

a and b were estimated by least squares methodology after applying the natural logarithm to both sides of the equation. The resulting values were: a = .08722864 and b = 1.17031155. The curve of {4} was an excellent fit to the data as R^2 =.9433. Figure 1 presents this curve and the 15 mid-interval estimates of the divorce intensity.

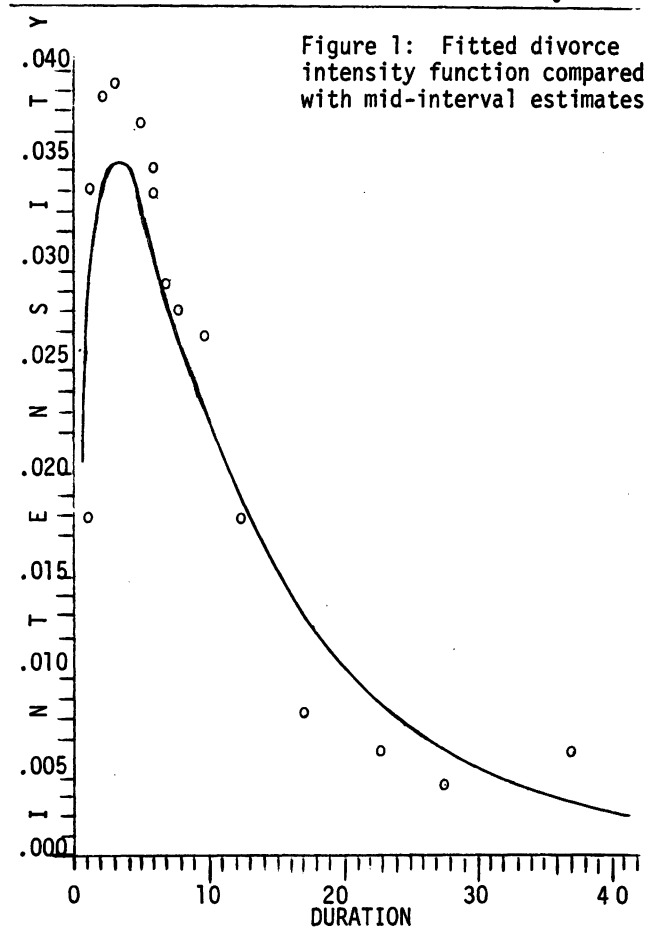

Figure 1: Fitted divorce intensity function compared with mid-interval estimates.

Table 2: From (3), 1971:

Groom's/Bride's Age at Time of Marriage	Percent Brides (Grooms) in Each Age Group						
	15-19	20-24	25-29	30-34	35-44	45-64	≥65
15-19	86.46(37.47)	12.72(54.38)	.60(6.48)	.15(1.07)	.07(.48)	0(.11)	0(.01)
20-24	39.41(4.67)	55.21(64.53)	4.34(22.98)	.71(5.15)	.29(2.24)	.04(.43)	0(0)
25-29	12.80(.81)	53.59(18.73)	25.51(40.37)	5.82(21.93)	2.06(14.84)	.22(3.22)	0(.10)
30-34	5.15(.44)	29.28(6.52)	33.79(19.65)	20.24(28.07)	10.23(33.76)	1.31(11.28)	0(.28)
35-44	2.12(.19)	11.68(2.25)	20.97(5.83)	22.32(11.85)	34.43(43.50)	8.41(34.94)	.07(1.44)
45-64	.50(0)	2.30(.28)	4.67(.64)	7.64(1.56)	28.38(11.01)	54.38(69.26)	2.13(17.18)
≥65	.19(0)	.14(0)	.62(0)	.75(0)	4.66(.56)	52.60(16.84)	41.04(82.66)

Table 3:

i	x_i	x_{i+1}	T_i	N_i	E_i	M_i	N_i'	L_i	D_i
1	0	1	35,144	2,174,642	7,080	6,875	2,162,035	14,314	.01644
2	1	2	67,232	2,162,998	33,826	14,238	2,114,934	15,062	.03242
3	2	3	72,580	2,112,331	92,691	28,615	1,991,025	15,563	.03729
4	3	4	67,232	2,006,415	151,405	41,963	1,813,047	15,636	.03795
5	4	5	57,300	1,887,581	199,612	54,188	1,633,781	15,550	.03588
6	5	6	48,896	1,823,164	238,147	67,358	1,517,659	15,861	.03293
7	6	7	45,076	1,755,331	259,827	80,123	1,415,381	16,118	.03256
8	7	8	36,672	1,712,248	277,431	93,878	1,340,939	16,579	.02791
9	8	9	31,324	1,609,081	282,756	103,802	1,222,523	16,408	.02614
10	9	10	28,268	1,552,582	290,980	115,990	1,145,612	16,660	.02517
11	10	15	103,140	7,556,332	447,585	818,250	6,290,497	90,905	.01665
12	15	20	68,760	7,660,747	1,091,589	1,300,757	5,268,401	127,322	.00665
13	20	25	53,480	8,979,748	1,426,867	2,187,817	5,365,064	133,026	.00507
14	25	30	27,504	8,295,415	1,879,514	2,680,387	3,735,514	156,015	.00377
15	30	45	19,864	18,865,575	4,678,455	10,064,267	4,122,853	544,484	.00517
			762,472	70,154,190	11,357,765	17,658,508	41,139,265	1,209,503	

Let $q(x_i, x_j) = \Pr\{\text{divorce } (x_i, x_j) \mid \text{no divorce } (.5, x_i)\} = 1 - \exp\left(-\int_{.5}^{x_j} \delta_x dx\right) / \exp\left(-\int_{.5}^{x_i} \delta_x dx\right)$ {5}

Since the integration required in {5} cannot be performed directly, the infinite series representation of e^x was employed; thus:

$$\exp\left(-\int_{.5}^{x_m} ax \exp(b\sqrt{x})dx\right) = \exp\left\{-a\sum_{K=0}^{\infty} \frac{b^k(x_m^{K/2+2} - .5^{K/2+2})}{K!\,(K/2+2)}\right\} \quad \{6\}$$

A computer program revealed that for each of the 15 x_i values, the sum in {6} converged after 50 terms ($e < 1 \times 10^{-11}$). This program also evaluated {5} for the 15 intervals shown in Table 3. The observed $q(x_i, x_j)$ values of Table 4 below were obtained using

$$q(x_i, x_j) = T_i/(N_i'' - E_i' - M_i') \quad \{7\}$$

where T_i is the number of divorces in 1971 among couples married $x_i - x_i$ years as of 1971, N_i'' is the number of couples married x_i years prior to 1971, E_i' is the number of marriages x_i years prior to 1971 ended by divorce prior to 1971, and M_i' is the number of marriages x_i years prior to 1971 ended by death of a spouse prior to 1971.

Table 4 provides a measure of the efficiency of equation {5} as a predictor of divorce. It is seen that the values obtained using equation {5} ($q(x_i, x_j)$ predicted) differ only slightly from the values obtained from {7} ($q(x_i, x_j)$ observed) with the exception of q(20,25). Equation {5} and the methodology of {6} may be used to obtain conditional probabilities of divorce not only for the x_i and x_j values of Table 4, but for intervals of any survival point and length.

Table 4: Predicted and observed values of $q(x_i, x_j)$ for selected intervals.

Interval x_i	Interval x_j	$q(x_i, x_j)$ Predicted	$q(x_i, x_j)$ Observed
0.5	1.0	0.011691	0.016255
1.0	2.0	0.030405	0.031789
2.0	3.0	0.033575	0.036454
4.0	5.0	0.032233	0.035072
6.0	7.0	0.028284	0.031847
8.0	9.0	0.024157	0.025622
10.0	15.0	0.083955	0.092281
15.0	20.0	0.055905	0.068918
20.0	25.0	0.037651	0.061111
25.0	30.0	0.025754	0.026938
30.0	45.0	0.039021	0.032978

The validity of the statistics used in this study may be examined by calculating the divorce rate per 1,000 married women from the figures of Table 3 and comparing it with the 1971 United States Office of Vital Statistics published figure. From Table 3, the number of divorces per 1,000 married women is 764,000/41,139.265 = 18 which is reasonably close to the published figure of 16.

Table 3 may also be used to estimate the percentage of marriages ending in divorce. There were 18,865,575 couples married 30-45 years prior to 1971. Among these couples, there were 4,678,455 divorces prior to 1971 plus an additional 19,864 during 1971. Although divorce data beyond 1971 was unavailable, we may project to future years assuming the 1971 number of divorces (19,864) among the 30-45 years married group will be constant throughout the period 1972-1985. The study period ends in 1985 as the couples married 30 years in 1971 approach 45 years of marriage in 1985. Furthermore, for each ensuing year, one fewer duration of marriage age is to be

tallied in projecting future divorces, i.e., each year removes another group from the count, since this group has exceeded the 45 year marriage duration, and by assumption, no divorces are possible beyond this period. Assuming a uniform distribution of divorce throughout the 15 year interval, 14/15 of 19,864 couples were divorced in 1972, 13/15 of 19,864 in 1973, ..., 1/15 of 19,864 will be divorced in 1985. In this manner, projected divorces totaled 139,048. Adding the 4,678,455 divorces prior to 1971, and the 19,864 in 1971, a total of 4,837,367 divorces have resulted. Dividing by 18,865,575, it is estimated that 25.64% of all marriages end in divorce.

Although not critical to the aforementioned results, there were a number of minor assumptions which were obviously incorrect. The divorce data was obtained assuming that for the entire divorce population, the percent divorces in any interval was the same as for the divorce registration area (sample). The Keats-Como mortality function was general (it included both men and women) and was based on the year 1970. The number of couples whose marriage ended by death would have been more accurate if separate male and female mortality functions had been used and if they were changed for each year in the study (1926-1971). The Groom-Bride and Bride-Groom age at marriage matrices (Table 2) were applied over the 45 year span of the study, yet these figures were based on 1971. For more accuracy, different matrices should have been used each year.

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